STATISTICAL AND MATHEMATICAL MODELS USED TO ESTABLISH THE COMPANY BANKRUPTCY RISK DIAGNOSIS

GÂDOIU MIHAELA[1]
[1] ***University of Piteşti, Piteşti, Romania***
mihaela_gadoiu@yahoo.com

Abstract: *The diagnosis of the bankruptcy risk is a very important one since it concerns the company's capacity to meet its exigible commitments both on the short, medium and long term. The study of the bankruptcy risk is complex, being carried out in terms of liquidity, as well as in terms of solvency of the company. The main statistical and mathematical models of diagnosis of the bankruptcy risk are: the Altman Z-score, in the version developed for the emerging markets and the Conan-Holder model. Among other famous models for the measurement of the bankruptcy risk there are the Central Bank's Balance Model and the BDFI model developed by the Central Bank of France, but only usable in some cases of French companies.*

Key words*: diagnostic, assessment, bankruptcy risk*

INTRODUCTION

Within the diagnosis of the company's bankruptcy risk, the analysis of the bankruptcy risk is very important, because it concerns the capacity of the company to meet its exigible commitments both on the short, medium and long term.

The specialised literature and practice has established three methods of assessing bankruptcy risk, namely the statistical analysis and, the dynamic analysis and the analysis based on statistical and mathematical methods.

The statistical analysis is used to diagnose the financial position of the company by means of the financial – patrimonial balance indicators.

The dynamic analysis of the company's bankruptcy risk proposes a differentiated approach, based on the net cash flows and on the company's self-financing capacity, which are viewed as essential indicators of a company's potential to meet the undertaken commitments.

The main **statistical and mathematical bankruptcy risk diagnosis models** are Altman's Z-score function, in the version developed for emerging markets, and the Conan-Holder model. Among other famous bankruptcy risk quantifying models, there are the Central Balance Sheet model, as well as the BDFI model, developed by Banque de France (the central bank of France), which can only be used for French company cases.

MATERIALS AND METHODS

This paper aims at highlighting the importance of the statistical and mathematical models used to diagnose bankruptcy risk. Starting from the idea that, in the financial theory and practice, very many approaches are used for the detection of the bankruptcy risk and that large part of those that are used are relevant only under certain specific and limitative conditions, the advantages of the statistical and mathematical bankruptcy risk models were highlighted. The research method used for the preparation of this paper implies, on the one hand, the theoretical substantiation of the notions related to the statistical and mathematical models used to assess company bankruptcy risk, and on the other hand, their transposition by means of a usage example for two companies.

RESULTS AND DISCUTIONS

In the context of the contemporary preoccupations for the development of statistical, mathematical and probabilistic models applicable in the economic field, it was obvious that company finances were a breeding ground for the exploration of these theoretic possibilities. Thus, since the 1960', there have been countless attempts to build models able to quantify the corporative bankruptcy risk, and a first remarkable achievement in this respect was the development of the Z-score function by the American professor Edward I. Altman in 1968 (in its original version).

The cornerstone of the model of classification proposed by Altman is the discriminant analysis, which allows for the development of new functional models for the classification of new observations in pre-established categories (classes) starting from data known in relation to the inclusion of other observation in these classes. The operating principle of the discriminant analysis is as follows: given that there us an initial set of data (for example, a lot of companies grouped in "bankrupt" and "non-bankrupt") and certain relevant characteristics concerning the qualification of the companies as "bankrupt" and "non-bankrupt", the discriminant analysis generates a classifier based on which decisions are made concerning the inclusion of the companies in one category or another. This classifier can be a function[1] or another probability measure[2].

The characteristics included in the analysis are related to indicators (financial rations) allowing for the assessment of the financial health of the company.

Altman's Z-score function is basically an application of the discriminant analysis with a linear classifier, which is a function with the following form:

$$f(i_1, i_2, \ldots, i_n) = x_0 + x_1 \cdot i_1 + x_2 \cdot i_2 + \ldots + x_n \cdot i_n \qquad (1),$$

where:

- $i_1, i_2, \ldots, i_n$ = indicators (financial rations) included in the analysis;
- $x_1, x_2, \ldots, x_n$ = indicator coefficients, quantifying the importance of each of them;
- x_0 = the free term of the linear function.

The value of the function calculated for each company separately allows for its inclusion in one of the two classes considered, and the calculation of the company's likelihood to go bankrupt.

The original model proposed by Altman (1968) is as follows[3]:

$$Z = 1{,}2 \cdot X_1 + 1{,}4 \cdot X_2 + 3{,}3 \cdot X_3 + 0{,}6 \cdot X_4 + 1{,}0 \cdot X_5 \qquad (2),$$

Where the financial ratios $X_i, i = \overline{1,5}$ are:

- $X_1 = \dfrac{\text{Current assets}}{\text{Total assets}}$
- $X_2 = \dfrac{\text{Retained earnings}}{\text{Total assets}}$

[1] In the specialized literature, such functions are called *score-functions*

[2] The probability of including loan applicants in one of the two above-mentioned categories.

[3] According to Altman (2002), page 14.

- $X_3 = \frac{\text{Earnings before interest and taxes (EBIT)}}{\text{Total assets}}$
- $X_4 = \frac{\text{Market value of equity}}{\text{Total liabilities}}$
- $X_5 = \frac{\text{Sales}}{\text{Total assets}}$

Based on the score recorded by individual companies three company classification "areas" were defined:

- $Z > 2.99$: the safety area. The bankruptcy risk of the company is low.
- $1{,}8 < Z < 2.99$: the uncertain area ("the grey area"). The company has a bankruptcy risk that cannot be ignored
- $Z < 1.8$: the hazardous area. High bankruptcy risk.

The Z-score function was developed based on a sample of 66 US manufacturing companies in the public sector, half of them being declared bankrupt. A question raised even before the Altman model was launched was related to the changes to be made to the model in order to be applied in the case of private companies as well[4]. The answer to this problem was the appearance of a modified version of the Z-score function under the following form:

$$Z' = 0{,}717 \cdot X_1 + 0{,}847 \cdot X_2 + 3{,}107 \cdot X_3 + 0{,}420 \cdot X_4 + 0{,}998 \cdot X_5 \qquad (3),$$

in which the financial forms used have the following meanings:

- $X_1 = \frac{\text{Current assets}}{\text{Total assets}}$
- $X_2 = \frac{\text{Retained earnings}}{\text{Total assets}}$
- $X_3 = \frac{\text{Earnings before interest and taxes (EBIT)}}{\text{Total assets}}$
- $X_4 = \frac{\text{Market value of equity}}{\text{Total liabilities}}$
- $X_5 = \frac{\text{Sales}}{\text{Total assets}}$

The critical areas are as follows:

- $Z' > 2.99$: the safe zone;
- $1.23 < Z' < 2{,}99$: the uncertain zone ("the grey zone");
- $Z' < 1.23$: the bankruptcy (distress) zone.

Subsequently, another reviewed version of the Altman model was published, which can be used in the case of non-industrial companies as well as in the case of companies in the emerging markets[5]:

$$Z'' = 6{,}56 \cdot X_1 + 3{,}26 \cdot X_2 + 6{,}72 \cdot X_3 + 1{,}05 \cdot X_4 \qquad (4),$$

[4] See Altman (2002), page 19.
[5] According to Altman (2002), page 22.

where:

- $X_1 = \frac{\text{Current assets}}{\text{Total assets}}$
- $X_2 = \frac{\text{Retained earnings}}{\text{Total assets}}$
- $X_3 = \frac{\text{Earnings before interest and taxes (EBIT)}}{\text{Total assets}}$
- $X_4 = \frac{\text{Book value of equity}}{\text{Total liabilities}}$

Depending on the Z-score values, in practice we can see the following situations:

- Z" > 2,6: safe zone;
- 1,1 < Z" < 2,6: the uncertain zone ("the grey zone");
- Z" < 1,1: the bankruptcy (distress) zone.

In relation to the performance of the Altman model, a number of empirical studies proved that it has an accuracy of more than 70% in the classification of companies.

Another linear classifier, similar to that generated by the Altman model, was developed by J. Conan and M. Holder in 1979 and is under the following form[6]:

$$z = 0{,}24 \cdot Y_1 + 0{,}22 \cdot Y_2 + 0{,}16 \cdot Y_3 - 0{,}87 \cdot Y_4 - 0{,}1 \cdot Y_5 \qquad (5),$$

where:

- $Y_1 = \frac{\text{Gross operating surplus (GOS)}}{\text{Total liabilities}}$
- $Y_2 = \frac{\text{Long-term capital}}{\text{Balance sheet total}}$
- $Y_3 = \frac{\text{Current assets - Stocks}}{\text{Balance sheet total}}$
- $Y_4 = \frac{\text{Financial expenses}}{\text{Net turnover}}$
- $Y_5 = \frac{\text{Staff costs}}{\text{Value added}}$

According to this model, a score of -0.21 corresponds to a 100% bankruptcy probability of the company, a score of 0.068 to a 50% bankruptcy probability, and a score of 0.164 to a 10% bankruptcy probability of the company.

Other famous bankruptcy risk quantifying models are the Central Balance Sheet model, and the BDFI model, developed by Banque de France (the central bank of France), but used only for French company cases [7].

Below, I calculated the Altman score (according to model (4), calibrated on emerging markets) and the Conan-Holder score for two trading companies, X and Y.

[6] See Stancu (2007), page 784.
[7] Idem 153.

The following table shows the results obtained for X S.A.:

Table 1

The Altman and Conan-Holder scores for S.C. X S.A. in the 2010 – 2013 period

YEAR	2010	2011	2012	2013
Altman score (Z")	5.4224	4.8074	6.2873	9.1724
Conan-Holder Score	0.1482	0.1040	0.1678	0.7235

Source: own calculations.

We can notice that, in relation to the Altman Z-score function, the results obtained by S.C. X S.A. placed the company in the safe zone each year, while the Conan-Holder model shows a low bankruptcy probability for the company, especially in the years 2012 and 2013.

For S.C.Y S.A. we will have:

Table 2

The Altman and Conan-Holder scores for S.C. Y S.A. in the 2010 – 2013 period

YEAR	2010	2011	2012	2013
Altman score (Z")	7.2096	5.3816	6.8603	6.3073
Conan-Holder Score	0.3512	0.2568	0.3424	0.2763

Source: own calculations.

The financial management of S.C. Y S.A. is characterized by balance and efficiency, which is highlighted by the scores obtained in the Altman and Conan-Holder models, which recommend it as a very safe company from the point of view of the bankruptcy risk.

The bankruptcy risk assessed based on the models proposed by Altman and Conan-Holder leads to the conclusion that the bankruptcy probability is extremely low, both in the case of S.C. X S.A., as well as in the case of S.C. Y S.A.

CONCLUSIONS

For the risk diagnosis, a static approach can be used (through the liquidity and solvency ratios), as well as a dynamic approach (through indicators such as long-term debt repayment ratio, the interest coverage ratio, the operating cash surplus coverage ratio, and others). The statistical analysis performed based on the liquidity and solvency ratios is a balance sheet analysis that does not take into account the operating dynamics and the economic values of a business. Although it is obvious that the balance sheet can only provide a static image of the financial situation of a company at a given time, the fact that the liquidity and solvency ratios are very easy to calculate makes them very popular in the modern financial analysis[8]. The use of the liquidity and solvency indicators is remarkable in terms of trend indicators when they are calculated for several periods of time. However, the ratio system analyzed cannot create an accurate image of the company's ability to pay to its creditors the interests and capital set based on contract[9].

These arguments prove that the decision to use only the statistical analysis and the dynamic analysis in determining the bankruptcy risk can be a fundamental error. It is

[8] See Helfert (2001), page 130.

[9] In the specialized literature, it is called *creditworthiness* (eng.).

recommended to use statistical and mathematical bankruptcy risk diagnosis models such as the Altman Z-score function, in the version prepared for emerging markets, and the Conan-Holder model.

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