

SIZING THE ENTITY'S TREASURY BY USING THE MATHEMATICAL MODELING

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Abstract: *Providing the economic balance represents a fundamental, sine qua non condition in order that economic entities to develop a both effective and stable activity. Beyond harmonizing the assets' liquidity and the liabilities' chargeability, a very important side of financial balance is that one regarding treasury's size optimization, as a concrete form of the favorable financial results manifestation. The present research focuses on the trial of optimizing the economic entities' treasury size by using several adequate mathematical models, such as: the Baumol model, the Miller-Orr model or the probabilistic model. As a research methodology there are used the review, the synthesis and the simulation.*

Key words: *financial balance, treasury, modeling, optimization, sizing.*

INTRODUCTION

By modeling, the decider may choose from optimization, under optimization or satisfaction, as principle for selecting the various decisions that are available to be taken into consideration. Optimization involves the selection of the best solution from the huge amount of possible solutions. Under optimization refers to solving a problem partially, to finding a solution for a sequence of a problem. Satisfaction refers rather to evaluating the importance of a problem, established by minimal values that might be taken by the objective function. These aspects are rediscovers in the situation of treasury decisions. There are various problems regarding the treasury administration and are characterized by strong particularities depending on the company's activity type, depending on the managerial style or on the financial information support. Mathematical modeling of the treasury administration should be treated distinctly depending on the size and quality of information in every moment of decision.

Thus, there are treasury decisions in certainty conditions, when information are complete; there are treasury decisions in uncertainty conditions, when information might be affected by side events that might or might not appear and, finally, there are treasury decisions in risk conditions, when information might be affected by side events that will surely occur with a certain probability.

MATERIALS AND METHODS

Numerical data and their target. In order to elaborate the present article, there are used various information such as: the monthly treasury demand for a 10 years period for a company or several indicators regarding the monthly treasury demand for that 10 years period.

By using these pieces of information, there are made several simulations by using the Baumol model, the Miller-Orr model or the probabilistic model in order to size the treasury amount so as to its value to be optimal.

The monthly treasury demand for the 10 years period is presented in the table below:

Table 1

Monthly Treasury Demand for a Ten Years Period

Year	Month	Treasury Demand	Year	Month	Treasury Demand	Year	Month	Treasury Demand	Year	Month	Treasury Demand	Year	Month	Treasury Demand
N	1	772	N+2	1	893	N+4	1	1155	N+6	1	980	N+8	1	844
	2	965		2	944		2	1089		2	951		2	1020
	3	1018		3	1064		3	976		3	1130		3	837
	4	1061		4	926		4	1164		4	1266		4	804
	5	809		5	991		5	1065		5	940		5	907
	6	1143		6	1075		6	856		6	983		6	1025
	7	1142		7	942		7	1217		7	974		7	1187
	8	1091		8	1064		8	944		8	1139		8	1102
	9	1009		9	1085		9	750		9	994		9	1053
	10	1046		10	978		10	912		10	829		10	828
	11	988		11	1025		11	964		11	963		11	839
	12	881		12	629		12	1134		12	883		12	972
N+1	1	987	N+3	1	1015	N+5	1	1425	N+7	1	1028	N+9	1	1098
	2	1069		2	1258		2	972		2	980		2	906
	3	946		3	980		3	770		3	980		3	1128
	4	884		4	1171		4	913		4	1098		4	1131
	5	1147		5	710		5	773		5	1304		5	1062
	6	847		6	974		6	977		6	940		6	1148
	7	1199		7	1056		7	998		7	884		7	1048
	8	967		8	820		8	880		8	1042		8	898
	9	976		9	1063		9	1064		9	795		9	788
	10	1087		10	1114		10	895		10	827		10	677
	11	955		11	1135		11	1230		11	1207		11	1130
	12	1057		12	814		12	959		12	960		12	993

Several indicators and several prevision assumptions related to the monthly values of treasury are presented in the tables below:

Table 2

Indicators Related to Monthly Treasury Demand for a Ten Years Period

Indicator	Value
Mean	995
Standard Error	12.43
Median	982
Mode	-
Standard Deviation	136.16
Sample Variance	18538
Kurtosis	33%
Skew	6%
Range	796
Minimum	629
Maximum	1425
Sum	119352
Count	120
Confidence Level(95.0%)	24.61

Table 3

**Prevision Assumptions Related
to Monthly Treasury Demand for a Ten Years Period**

Prevision Assumption	Value
Yearly Mean Demand	99
Penury Cost	5
Interest Rate	15%
Administration Cost	2
Mean Payments for the First Day of the Month	15

Mathematical models: In order to optimize the treasury size, there are used three mathematical methods, namely the Baumol model, the Miller-Orr model or the probabilistic model.

The Baumol model is a mathematical model that comes from Wilson-Whitin model of optimizing the stock amount. However, it is adapted to focus on the treasury demand and on the necessity of having a minimal level of liquidities in order to engage a minimal level of costs. It can be applied when sizing the treasury in certainty conditions.

The Baumol model has several work hypothesis, such as:

- Cashing and payments are well-known;
- Buying or selling of investments have direct costs, considered to have fixed amounts;
- Cash inputs are discontinuous while cash outputs are continuous;
- Opportunity cost of cash is constant.

There are two possible situations, namely a rhythmical reduction of cash and an no rhythmical reduction of cash.

If cash reduction is rhythmical, the intervals between two transactions are equal and the spent amount cash is constant for a time period. In this case, the minimal level of cash and its cost are revealed by the following relations:

$$n_{optimal} = \sqrt{2 \cdot \frac{N \cdot c_t}{D \cdot i}},$$

$$C_{optimal} = C(n_{optimal}) = \sqrt{2 \cdot N \cdot c_t \cdot i}.$$

If cash reduction is no rhythmical, the intervals between two transactions are unequal and appear the possibility of stock disruption. In this case, the minimal level of cash and its cost are revealed by the following relations:

$$n_{optimal} = \sqrt{2 \cdot \frac{N \cdot ch_{tr}}{D \cdot i}} \cdot \sqrt{\frac{i + c_p}{c_p}},$$

$$C_{optimal} = C(n_{optimal}, s_{optimal}) = \sqrt{2 \cdot N \cdot D \cdot i \cdot ch_{tr}} \cdot \sqrt{\frac{c_p}{i + c_p}}.$$

There were used the following notations: D - the period being analyzed; i - the interest rate; N - total level of demand; ch_{tr} - transaction expense; c_p - penalty cost; s - period without stock penury.

The Miller-Orr model is a real alternative for the Baumol model.

The Miller-Orr model's work hypotheses are:

- The medium variations of the daily cash-flow amount is 0;
- Buying and selling of investments determine fixed costs on each transaction;
- Cash incomes and outputs have a random statistical distribution;
- The transactions involving investments are instant.

By specific methods of nonlinear programming, minimization of cash costs has been achieved by assuming the work hypothesis of forecasting the cash outputs level for a certain time period. Also, it involves an arbitration process between the total transactional and administration costs and total option costs. Thus, it is generated the difference calculating relation:

$$Difference = 3 \cdot \sqrt[3]{\frac{\frac{3}{4} \cdot \sigma^2 \cdot c_a}{c_d}}$$

Once determined the optimal spread, it is necessary to determine the superior and inferior limits of the model. The inferior limit is previously established by the decider and it represents a minimal accepted cash threshold for the end of the day. The superior limit is determined using the following relation:

$$L_{sup\ erior} = L_{inf\ erior} + \frac{Ecart}{3}$$

The decision rule in case of using this model is quite simple. As long as treasury level is beyond the amount of Z^2 cash units, the cash-flows accumulate in the cash amount of company, in order to avoid transactional costs. But, from the point where the cash amount is situated over this difference, the company's administrator will place the cash surplus in investments until the cash amount will become again of Z units.

The probabilistic model is used in order to deliver values situated closer to the random behavior of cash variations. This model should be treated separately depending on the form of the random variable. If the random variable, representing the cash demand L , takes plenty of finite values, it is considered as being a discreet random variable while if it takes plenty of infinite values, it is considered as being a continuous random variable.

The model's solution represents, in fact, the determination of the optimal cash level S_0 which respects the double inequality $M[C(S_0 - 1)] \geq M[C(S_0)] \geq M[C(S_0 + 1)]$, where M represents the medium value of the random variable that has been analyzed.

By virtue of this information, the double inequity reduces to the practical relation of determining the optimum of cash value S_0 , namely: $P(X \leq S_0 - 1) \leq \frac{c_p}{c_d + c_p} \leq P(X \leq S_0)$,

where P represents the repartition function of the random variable cash demand X . The average total cost of possession or lack of cash is determined by the relation:

$$C(S_0) = c_d \cdot \sum_{r=0}^{S_0} (S_0 - r) \cdot p(r) + c_p \cdot \sum_{r=S_0+1}^{\infty} (r - S_0) \cdot p(r).$$

The main objectives for this study are:

- Estimating the optimal size of treasury by using the three models comparatively;
- Estimating the treasury demand fluctuations during a certain time period;
- Measuring the cash risk;
- Placing the cash exceeding.

RESEARCH RESULTS

The research is based on an Excel working interface that applies the Baumol model, the Miller-Orr model and the probabilistic model on the information presented before.

Thus, the result of implementing the Baumol model is:

Table 4

The Baumol model results

Unit Supply	364
Optimal Period	4
Optimal Number	3
Treasury Cost	15
Optimal Treasury	995

In the same time, the result of implementing the Miller-Orr model is:

Table 5

The Miller-Orr model results

Minimal Treasury	15
Optimal Difference	293
Superior Level	113

Not at last, the result of implementing the probabilistic model is:

Table 6

The probabilistic model results

Sub unitary Interval	101		Cost Report	0.93			
Treasury Intervals							
Inferior	Superior	Middle of the Interval	Absolute Frequency	Relative Frequency	Cumulated Frequency	Test	
629	730	679	1	0.83%	0.83%	0	15120
730	830	780	2	1.67%	2.50%	0	12090
830	931	881	13	10.83%	13.33%	0	9060
931	1032	981	18	15.00%	28.33%	0	6060
1032	1132	1082	40	33.33%	61.67%	0	3030
1132	1233	1183	27	22.50%	84.17%	1183	0
1233	1334	1284	15	12.50%	96.67%	0	2
1334	1435	1384	3	2.50%	99.17%	0	1
1435			1	0.83%	100.00%	0	35490
			120			1183	
Optimal Treasury	1183						
Treasury Cost	80853						

Measuring the cash risk, it is generated the following situation:

Table 7

The measuring of cash risk

Baumol	0.000					
Miller-Orr	0.000	Signification Level	5.00%		Level of Treasury Demand	1000
Probabilistic demand	0.11%	Z Coefficient	1.96			
		Trust Interval	972	1017	Probability of Penury Occurrence	0.29%

CONCLUSIONS

Corroborating the information presented in the previous section, the treasury demand has an evolution that could be very well represented by the following chart:

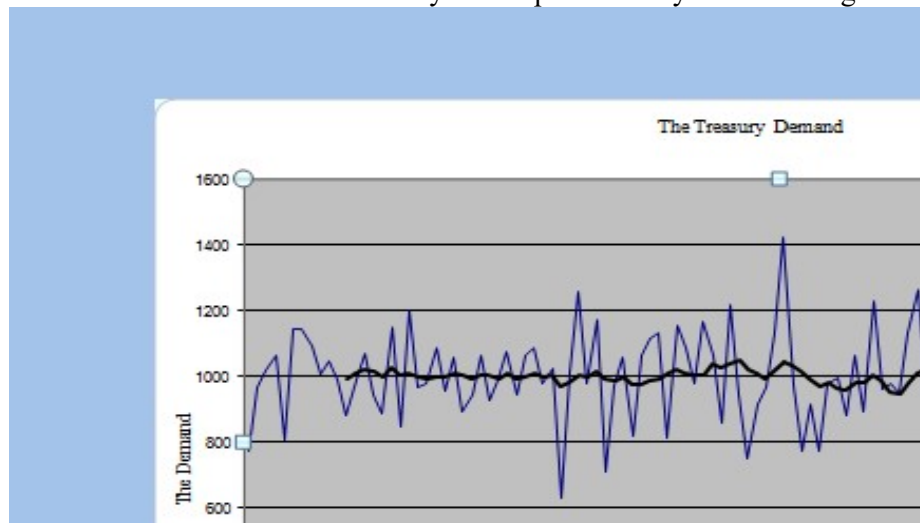


Fig. 1: The evolution of the treasury demand

A positive sold of treasury describes an expected financial situation and the best solution for rousing it is investments. But those investments are really profitable that respect two conditions: increased profitability and minimum risk.

In the present case, an optimal pack of investments would include 5 types of investments; with a profitability of about 25% and a risk of about 20%. The optimal structure of the pack is obvious from the following chart:

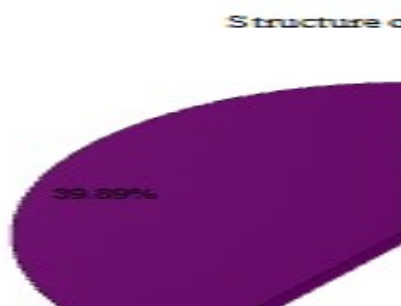


Fig. 2: The structure of the investments pack

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