THE USE OF THE BOX-COX TRANSFORMATION, AN ALTERNATIVE TO THE QUADRATIC PRODUCTION FUNCTIONS

CIPRIAN IOAN RUJESCU [1], RAMONA CIOLAC [1], SIMONA CRISTINA MARTIN [1], MARIUS BUTUR [2]

[1]Banat`s University of Agricultural Sciences and Veterinary Medicine "King Michael I of Romania" from Timisoara;
rujescu@usab-tm.ro
[2]Victor Babes University of Medicine and Pharmacy, Timisoara, ROMANIA

Abstract: *This study pleads for the use of other alternative variants, flexible from the point of view of the mathematical expression, for the production functions in the case of determining the influence of fertilizing substances on cereal production. Practically, we examine the differences from the quadratic function, corresponding to a real situation of corn fertilization for a production function obtained by the Box–Cox transformation. The values obtained are very close, but in the context of a more flexible expression, they indicate the possibility of use for fertilization optimization issues.*

Key words: *agriculture, optimization*

INTRODUCTION

The Box-Cox is based on the equation

$$y_i^{(\lambda)} = \begin{cases} \dfrac{y_i^{\lambda} - 1}{\lambda}, & \lambda \neq 0 \\ \log y_i, & \lambda = 0 \end{cases}.$$

This appears as a modification of a previous expression introduced by Tukey (Sakia 1992, Box, G. E. P. & Cox, D. R., 1964). In general, the influence that fertilizer doses applied on cereal crops have on agricultural production is modelled mathematically by using second degree functions. The improvement of models, by which we understand a closer similarity to the real phenomenon, can be performed by means of the Box-Cox transformation (Vanoti, M.B. – 1994, Loren W. Tauer – 2000).

The purpose of this study is to analyze a specific example, by examining the differences between the quadratic and the Box-Cox transformation.

MATERIAL AND METHODS

In the specific case concerning the influence of various chemical fertilizer doses applied on the Minhybrid 511 Fundulea corn, performed on the medium levigated Chernozem at I.C.C.P.T. Fundulea, on productivity (table 1) [Marian T. 1976], for a regression function with the form $y = ax^2+bx+c$, we determined the a, b and c coefficients in the system:

$$\begin{cases} a\sum_{i=1}^{n} x_i^{\,4} + b\sum_{i=1}^{n} x_i^{\,3} + c\sum_{i=1}^{n} x_i^{\,2} = \sum_{i=1}^{n} x_i^{\,2} y_i \\ a\sum_{i=1}^{n} x_i^{\,3} + b\sum_{i=1}^{n} x_i^{\,2} + c\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i \\ a\sum_{i=1}^{n} x_i^{\,2} + b\sum_{i=1}^{n} x_i + cn = a\sum_{i=1}^{n} y_i \end{cases}$$

by replacing the experimental data,

$$\begin{cases} a \cdot 1.030625 \cdot 10^{10} + b \cdot 3.9375 \cdot 10^{7} + c \cdot 1.625 \cdot 10^{5} = 1.205 \cdot 10^{7} \\ a \cdot 3.9375 \cdot 10^{7} + b \cdot 1.625 \cdot 10^{5} + c \cdot 750 = 5.53 \cdot 10^{4} \\ a \cdot 1.625 \cdot 10^{5} + b \cdot 750 + 5 \cdot c = 351 \end{cases}$$

$$\Delta = \begin{vmatrix} 1.030625 \cdot 10^{10} & 3.9375 \cdot 10^{7} & 1.625 \cdot 10^{5} \\ 3.9375 \cdot 10^{7} & 1.625 \cdot 10^{5} & 750 \\ 1.625 \cdot 10^{5} & 750 & 5 \end{vmatrix}$$

$$\Delta_a = \begin{vmatrix} 1.205 \cdot 10^{7} & 3.9375 \cdot 10^{7} & 1.625 \cdot 10^{5} \\ 5.53 \cdot 10^{4} & 1.625 \cdot 10^{5} & 750 \\ 351 & 750 & 5 \end{vmatrix}$$

$$\Delta_b = \begin{vmatrix} 1.030625 \cdot 10^{10} & 1{,}205 \cdot 10^{7} & 1.625 \cdot 10^{5} \\ 3.9375 \cdot 10^{7} & 5.53 \cdot 10^{4} & 750 \\ 1.625 \cdot 10^{5} & 351 & 5 \end{vmatrix}$$

$$\Delta_c = \begin{vmatrix} 1.030625 \cdot 10^{10} & 3.9375 \cdot 10^{7} & 1.205 \cdot 10^{7} \\ 3.9375 \cdot 10^{7} & 1.625 \cdot 10^{5} & 5.53 \cdot 10^{4} \\ 1.625 \cdot 10^{5} & 750 & 351 \end{vmatrix}$$

The solutions of this system are:

$$a = \frac{\Delta_a}{\Delta} = -0.0003,\; b = \frac{\Delta_b}{\Delta} = 0.1401,\; c = \frac{\Delta_c}{\Delta} = 58.62,$$

so the production function is represented by the expression:

$y = -0.0003\, x^2 + 0.1401\, x + 58.62.$

Table 1

Fertilization, kg/ha	Productivity, q/ha
0	59
100	69
150	72
200	77
300	74

By using the Box-Cox transformation, the quadratic model becomes:

$$(1)\quad \frac{y^{\lambda} - 1}{\lambda} = a\left(\frac{x^{\lambda} - 1}{\lambda}\right)^{2} + b\left(\frac{x^{\lambda} - 1}{\lambda}\right) + c$$

where y represents the cereal production obtained after assigning factor x (e.g. nitrogen fertilizers), with the a, b, c, λ parameters. Let us notice that for λ=1 we obtain a second degree function.

RESULTS AND DISCUSSIONS

Experimental results in table 1 are transformed as follows:

Table 2

$X = \frac{x^{\lambda}-1}{\lambda}$	$Y = \frac{y^{\lambda}-1}{\lambda}$
-1.333	27.051
40.830	30.587
55.815	31.622
69.577	33.324
94.779	32.307

By using the least squares method we determine the coefficients:

$a = -0.000682,\ b = 0.121327, c = 27.255007$.

Then,

$$(2)\quad \frac{y^{\lambda}-1}{\lambda} = a\left(\frac{x^{\lambda}-1}{\lambda}\right)^2 + b\left(\frac{x^{\lambda}-1}{\lambda}\right) + c \Rightarrow y = \left(\lambda\left(a\left(\frac{x^{\lambda}-1}{\lambda}\right)^2 + b\left(\frac{x^{\lambda}-1}{\lambda}\right) + c\right) + 1\right)^{1/\lambda}.$$

With the data above, for the value λ=0.75 in equation (2) we have:

$$y = \left(0.75\cdot\left((-0.000682)\cdot\left(\frac{x^{0.75}-1}{0.75}\right)^2 + 0.121327\cdot\left(\frac{x^{0.75}-1}{0.75}\right) + 27.255007\right) + 1\right)^{1/0.75}$$

The table below presents the results corresponding to the double-digit values of x (nitrogen fertilizer doses) within the interval (100,180). The differences between the two models are slim, on the fertilization levels indicated, the obtained production indicating small variations, below 1 kg/ha.

Table 3

x	y_1 (II degree function)	y_2 (Box-Cox function)
100	70,39	69,63
110	70,97	70,40
120	71,50	71,11
130	71,98	71,76
140	72,41	72,35
150	72,81	72,88
160	73,17	73,35
170	73,49	73,76
180	73,78	74,11

CONCLUSIONS

Differences have no values implying important modifications from a quantitative perspective of production; this happens because, in agricultural practice, their size of about 1 kg/ha can be considered negligible.

Although the values obtained are similar, the equation obtained by means of the Box-Cox transformation has a flexible expression and the values of parameter λ allow an adjustment of the model according to the specific case under study.

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