THE USE OF A TWO-FACTOR MODEL FOR THE FERTILIZATION OF AGRICULTURAL CROPS IN ORDER TO DETERMINE THE OPTIMAL PRODUCTION AREA

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Abstract: *The model uses a two-factor function to connect production and its determining factors, with the form* $Q = aN^2 + bN + cP^2 + dP + eNP + f$ *and the optimal area is determined with the equation* $\left(N_{optim} \cdot p_N + P_{optim} \cdot p_P + s_f\right) \cdot s_{optim} = S$ *where* N_{optim} *and* P_{optim} *represent the economically optimal quantities where fertilizers are used;* p_N *and* p_P *represent their unit prices,* s_f *the fixed sums associated to the area unit;* s_{optim} *is the optimal surface to be determined starting from the value S of the available capital. Specific calculations have been performed starting from a production function for two-row barley, fertilized with nitrogen and phosphorus. Calculations performed are consistent with the product values of the year 2013 in Romania.*

Key words: *agriculture, optimization*

INTRODUCTION

The present work represents an extension of the authors' previous study and consists in using a more complex model, with a two-factor function representing the connection between two factors considered dominant, the quantities of nitrogen and phosphorus and the crop production. The case will be extended to a production function with the form $Q = aN^2 + bN + cP^2 + dP + eNP + f$, the restriction being given by an initial fixed investment, assuming that no minimum production value is imposed. The optimal production area will be determined for the value S of the available capital, by using the equation $\left(N_{optim} \cdot p_N + P_{optim} \cdot p_P + s_f\right) \cdot s_{optim} = S$. N_{optim} and P_{optim} represent the optimal economic values (the maximum differences between the points situated on the production function graph and the costs), p_N and p_N the unit price of fertilizers and s_{optim} the optimal area [12]. The fixed sums s_f consisted of the fixed costs for the unit area and the associated taxes, but also took into account the subsidies granted per unit area used for agricultural production. The purpose of this study is to determine the optimal areas resulting after the iteration with the various given values of an initial investment, by using the model with the described two-factor functions.

MATERIAL AND METHOD

The framework optimization model is differential, and consists in determining the maximum point of a real function of two real variables given by the difference between income and costs [1,9].

The profit equation becomes:

$$(1)\ \Pr = f(k)\left(aN^2 + bN + cP^2 + dP + eNP + f\right) - p_N N - p_P P - s_f$$

where we considered that the selling price of the obtained products varies and depends on the quantity existing on the market; this dependence is expressed by the relationship: p = f(k).

We cancel the first-order partial derivatives and obtain:

$$\begin{cases} \dfrac{\partial \Pr}{\partial N} = 2aNf(k) + bf(k) + ePf(k) - p_N = 0 \\ \dfrac{\partial \Pr}{\partial N} = 2cPf(k) + df(k) + eNf(k) - p_P = 0 \end{cases}$$

meaning the system of equations:

$$\begin{cases} 2af(k)N + +ef(k)P = p_N - bf(k) \\ ef(k)N + 2cf(k)P = p_P - df(k) \end{cases}.$$

For the resolution, we have the system determinant:

$$\Delta = \begin{vmatrix} 2af(k) & ef(k) \\ ef(k) & 2cf(k) \end{vmatrix}$$

$$\Delta = -(e^2 - 4ac)f^2(k)$$

and the unknown quantities of the system correspond to:

$$\Delta_N = \begin{vmatrix} p_N - bf(k) & ef(k) \\ p_P - df(k) & 2cf(k) \end{vmatrix}$$

$$\Delta_N = 2 \cdot cP_N \cdot f(k) - 2 \cdot c \cdot b \cdot f^2(k) - e \cdot p_P \cdot f(k) + d \cdot e \cdot f^2(k)$$

$$\Delta_N = f(k)(2 \cdot c \cdot p_N - e \cdot p_P + d \cdot e \cdot f(k) - 2 \cdot b \cdot c \cdot f(k))$$

for N, and:

$$\Delta_P = \begin{vmatrix} 2a \cdot f(k) & p_N - b \cdot f(k) \\ e \cdot f(k) & p_P - d \cdot f(k) \end{vmatrix}$$

$$\Delta_P = f(k) \cdot (2 \cdot a \cdot p_P - e \cdot p_N + b \cdot e \cdot f(k) - 2 \cdot a \cdot d \cdot f(k))$$

for P. Then the optimal values for N and P will be:

$$N_{optim} = \frac{2 \cdot c \cdot p_N - e \cdot p_P + d \cdot e \cdot f(k) - 2 \cdot b \cdot c \cdot f(k)}{-f(k)(e^2 - 4ac)}$$

$$P_{optim} = \frac{2 \cdot a \cdot p_P - e \cdot p_N + b \cdot e \cdot f(k) - 2 \cdot a \cdot d \cdot f(k)}{-f(k)(e^2 - 4ac)}.$$

So the maximum profit for the unit area is:

$$\Pr = f(k)\left(aN_{optim}{}^2 + bN_{optim} + cP_{optim}{}^2 + dP_{optim} + eN_{optim}P_{optim} + f\right) - $$
$$- p_N N_{optim} - p_P P_{optim} - c_f$$

where N_{optim} and P_{optim} will be computed by means of the above equations.

The optimal crop area is determined by the equation:

$$\left(N_{optim} \cdot p_N + P_{optim} \cdot p_P + s_f\right) \cdot s_{optim} = S$$

consequently

$$s_{optim} = \frac{S}{N_{optim} \cdot p_N + P_{optim} \cdot p_P + s_f}$$

hence

(2) $$s_{optim} = \frac{S}{\dfrac{2cp_N - ep_P + def(k) - 2bcf(k)}{-f(k)(e^2 - 4ac)} \cdot p_N + \dfrac{2ap_P - ep_N + ebf(k) - 2adf(k)}{-f(k)(e^2 - 4ac)} \cdot p_P + s_f}$$ [12]

RESULTS AND DISCUSSIONS

Let us consider now the specific example with the production function

$$y = -0{,}139N^2 + 28{,}206N - 0{,}074P^2 + 19{,}946P - 0{,}016NP + 780{,}51$$

indicating the connection between the fertilizer doses with N and P and the production of two-row barley. The data for the year 2013 indicate the price of ammonium nitrate fertilizers 33.4% is around 331.8 euro/ton, and for superphosphate 309.1 euro/ton; the fixed sums for a hectare of two-row barley were 220 euro and were obtained by adding up the fixed costs and the corresponding taxes, and finally taking into account the subsidized value. We assume that the price of two-row barley is known, f(k)=186 euro/t. (**INS -** National Institute of Statistics, **Fiscal Code, APIA -** Payments and Intervention Agency for Agriculture, **ANAF –** The National Tax Agency).

The above-mentioned data, replaced in equation 2, lead to the following result:

$$s_{optim} = \frac{S}{88{,}48 \cdot 0{,}3318 + 113{,}97 \cdot 0{,}3091 + 220} = 0{,}003513S$$

Starting from a few values of the available capital, we obtain the data in the table below:

Available S (euro)	Optimal area (ha)
30000	105,39
35000	122,955
40000	140,52
45000	158,085
50000	175,65

The size of the cultivated land cannot be chosen randomly; moreover, the optimization calculations become mandatory especially under the contractual terms specific to the financial support measures for the beneficiaries of the European Agricultural Fund for Rural Development or the like, where capital must be strictly used to the purpose of setting up and supporting agricultural crops.

CONCLUSIONS

The simulation can also be performed for other crops or it can be iterated for other production functions. Although it is a well-known fact that the physiological indicators of plant development are influenced by an extensive series of factors, not all of them have a significant economic effect. The use of additional factors for the optimization studies will bring the phenomenon under study closer to reality. It is advisable to correlate these aspects with the complexity of calculations resulted by analyzing new factors, but also with the interest for a certain degree of computational precision.

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